Logique monadique du deuxième ordre et sémantiques concurrentes des réseaux de Petri.

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Modeling systems behaviours by words

Subsets of finitely generated free monoids [Kleene][Büchi].. :

MSO definability $\Leftrightarrow$ finite automata acceptance

$\Leftrightarrow$ rational expressions $\Leftrightarrow$ algebraic recognizability.

Logics $\longrightarrow$ *specify properties* : MSO subsumes linear and branching temporal logics

Characterize MSO logics in terms of

Automata , machines $\longrightarrow$ recognizability, realizability

Algebraic structure $\longrightarrow$ syntactic expressions, algebraic properties

Modeling concurrent/distributed behaviours by labelled partial orders (pomsets): expressivity, accuracy.

The framework of words "extended" to

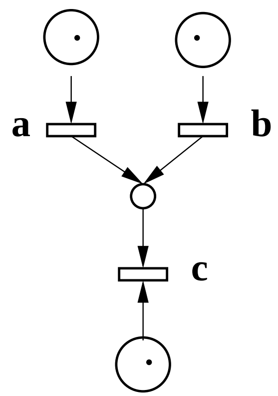
Mazurkiewicz traces [Thomas] [Ochmanski] [Zielonka]..,

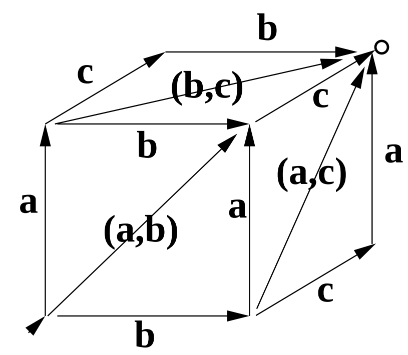
Message Sequence Charts [Henriksen&all] [Alur&all] [Morin]..,

Series-parallel pomsets [Lodaya,Weil] [Kuske].

Strong results on *Pomsets without autoconcurrency* [Droste, Gastin, Kuske], *Labelled dags without autoconcurrency ($\Sigma - C$ -Dags)* [Bollig,Leucker], *subsets of Concurrency Monoids* [Droste, Kuske] *Consistent sets of pomsets* [Arnold][Morin]. *Graphs*: MSO definability vs Algebraic recognizability [Courcelle], EMSO definability vs Graph acceptors [Thomas],

Step Transition Systems and Petri Nets





Step Transition Systems (STS): general model of concurrent systems, (step) marking graphs of Petri nets [Mukund], Distributed TS [Lodaya&all].

Deterministic STS without autoconcurrency are Local Trace Languages [Kleijn&all], [Thiagarajan&all].

Contribution

We consider STS with multisteps (autoconcurrency).

Define a natural STS pomset semantics and characterize the resulting languages for Deterministic STS.

Characterize the MSO definability of DSTS languages in terms of regularity and finite DSTS languages.

Induced properties for both "anonymous tokens" and "individual tokens" Petri nets semantics.

Rely on previous work on *Local Trace pomsets* [Kuske, Morin] and *Regular sets of pomsets* [Fanchon, Morin].

A scheme for DSTS languages.

Theorem: Let $\mathcal{L}$ be the pomset language of a (finitely branching) deterministic STS then the following items are equivalent:

$\mathcal{L}$ is MSO definable

$\mathcal{L}$ is the language of a finite DSTS

$\mathcal{L}$ is *regular*

The *base* $B(\mathcal{L})$ is MSO definable

$B(\mathcal{L})$ is regular and *prime bounded*

The set of *step extensions* $SE(\mathcal{L})$ is MSO definable

$SE(\mathcal{L})$ is regular

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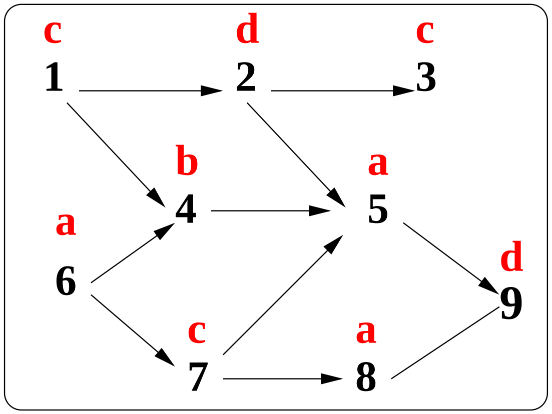
Pomsets

Σ finite alphabet , a Σ-labelled partial order is a tuple $(E, \preceq, l)$:

$(E, \preceq)$ is a finite partial order (events), $l : E \to \Sigma$ is a labelling

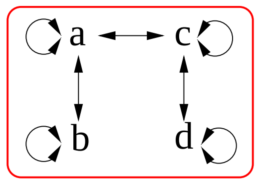
$\lessdot$ "prime intervals" of $\preceq$: $e \lessdot e' \Leftrightarrow [e \preceq e' \bigwedge (e \preceq e'' \prec e' \Rightarrow e = e'')]$

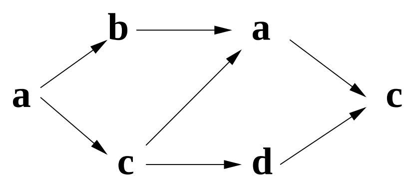


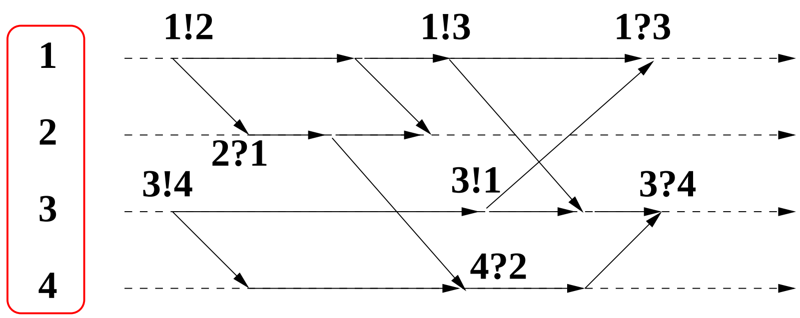
A labelled poset

Pomsets ($\mathbb{P}(\Sigma)$): isomorphism classes of LPO, denoted by any class member.

Mazurkiewicz traces, Message sequence charts







Pomsets

$$t = (E, \preceq, l) \in \mathbb{P}(\Sigma),\ e, e', e" \in E$$

co concurrency relation : $e\, co\, e' \Leftrightarrow \neg(e \leq e') \bigwedge \neg(e' \leq e)$

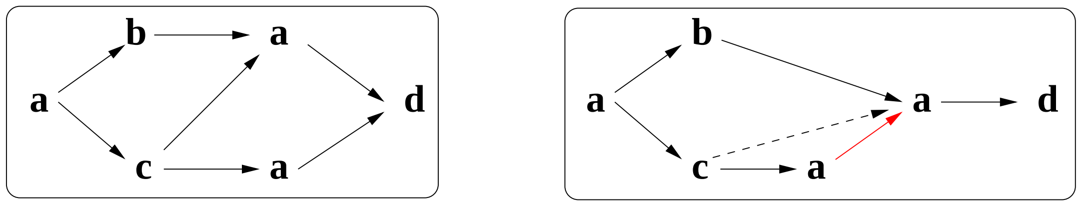
Width of t: maximal size of a cut

Autoconcurrency : $\exists e, e' \in E. e\, co\, e' \bigwedge l(e) = l(e')$

Strong concatenation : $t.t'$

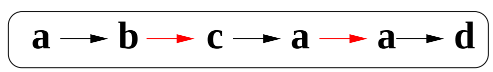
Parallel composition : $t \parallel t$

Aspects of pomsets



A pomset t and an order extension of t

A step extension of t and its multiset sequence representation.



A linear extension of t.

Representations of sets of pomsets

Order Extensions $OE(\mathcal{L})$

$$OE(E, \preceq, l) = \{(E, \preceq', l) \in \mathbb{P}(\Sigma) \mid \preceq \subseteq \preceq'\}$$

$$\mathcal{L} \subseteq \mathbb{P}(\Sigma) \text{ is weak if } \mathcal{L} = OE(\mathcal{L})$$

Linear Extensions $LE(\mathcal{L})$

$$\Sigma^* = \{(E, \preceq, l) \in \mathbb{P}(\Sigma) \mid \forall e, e' \in E : \neg(e\, co\, e')\}$$

$$LE(\mathcal{L}) = OE(\mathcal{L}) \cap \Sigma^\star$$

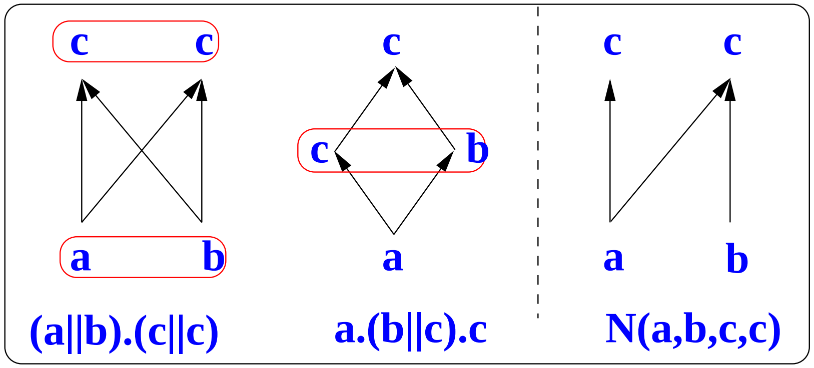
Step Extensions $SE(\mathcal{L})$

$$\mathbb{S}(\Sigma) = \{(E, \preceq, l) \in \mathbb{P}(\Sigma) \mid \forall e, e', e'' \in E : e\, co\, e'\, co\, e'' \Rightarrow e\, co\, e''\}$$

$$SE(\mathcal{L}) = OE(\mathcal{L}) \cap \mathbb{S}(\Sigma)$$

$M(\Sigma)$ multisets on Σ: $(\mathbb{S}(\Sigma), .)$ and $M(\Sigma)^\star$ are isomorphic monoids.

Step Extensions



$$SE(N(a,b,c,c)) = SE(\{(a||b).(c||c), a.(b||c).c\})$$

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FO and MSO Logics of pomsets

x, y 1st order variables denote events , X, Y 2nd order variables denote sets of events (monadic)

Atoms related to labelling and ordering of events : $P_a(x)$, $x \lessdot y$

Formulae of $FO(\Sigma, \lessdot)$

$$\varphi := P_a(x) \mid x \lessdot y \mid x = y \mid \varphi \bigwedge \varphi \mid \neg\varphi \mid \exists x.\varphi$$

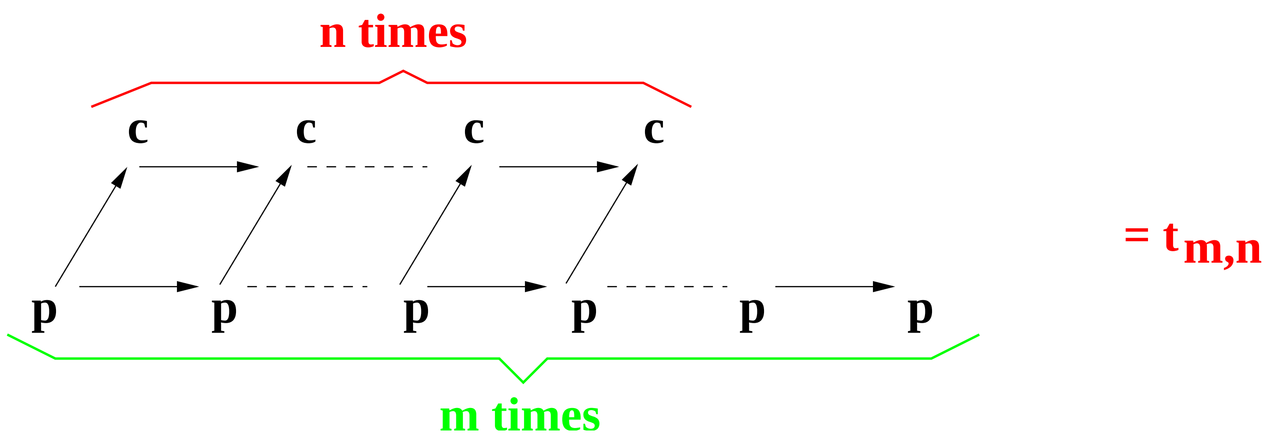
Formulae of $MSO(\Sigma, \lessdot)$

$$\varphi := P_a(x) \mid x \lessdot y \mid x = y \mid \varphi \bigwedge \varphi \mid \neg\varphi \mid \exists x.\varphi$$

$$x \in X \mid \exists X.\varphi$$

$\mathcal{L} \subseteq \mathbb{P}(\Sigma)$ is FO/MSO definable iff it exists an FO/MSO formula φ s.t. $\mathcal{L} = \{u \in \mathbb{P}(\Sigma) \mid t \models \varphi\}$.

Definable sets of pomsets



$\mathcal{L} = \{t_{m,n}\}$ is $FO(\Sigma, \preceq)$ definable:

$$[\forall x, y. P_p(y) \bigwedge x \lessdot y \Rightarrow P_p(x)] \bigwedge [\forall x.(P_c(x) \Rightarrow \exists y. y \lessdot x \bigwedge P_p(y))] \bigwedge \ldots.$$

$LE(\mathcal{L}) = \{u \in \Sigma^\star : \forall v.(\exists w : u = v.w) \Rightarrow |v|_c \leq |v|_p\}$ is not regular, thus not $MSO(\Sigma, \preceq)$ definable

Logics of pomsets

$\lessdot$ is FO definable from $\preceq$

$$x \lessdot y \equiv [x \preceq y \bigwedge \forall z.(x \preceq z \prec y \Rightarrow x = z)]$$

$\preceq$ is MSO definable from $\lessdot$

$$x \preceq y \equiv \forall X.[\forall v, z.v \in X \bigwedge v \lessdot z \Rightarrow z \in X] \bigwedge x \in X \Rightarrow y \in X$$

Prefixes (ideals), cuts and chains are MSO definable

$$Pref(X) \equiv \forall x, y.x \in X \bigwedge y \lessdot x \Rightarrow y \in X$$

$$Cut(X) \equiv \forall x, y.(x \in X \bigwedge y \in X \Rightarrow x\, co\, y) \bigwedge (\forall z.z\, co\, X \Rightarrow z \in X)$$

$$Chain(X) \equiv \forall x, y \in X.[x \lessdot y \bigvee y \lessdot x \bigvee (\exists z \in X.x \lessdot z \leq y \bigvee y \lessdot z \leq x)]$$

Modalities of many partial order temporal logics are FO/MSO definable.

Local LTL :e is an event of t

$$t, e \models \exists < a > .\varphi$$

$$[\exists < a > .\varphi](x) \equiv \exists y. x \lessdot y \wedge P_a(y) \wedge \varphi(y)$$

$$t, e \models \varphi \, Until \, \psi$$

$$[\varphi \, Until \, \psi](x) \equiv \exists y. x \prec y \wedge \psi(y) \wedge \forall z. x \preceq z \prec y \Rightarrow \varphi(z)$$

Global LTL :C is a prefix of t

$$t, C \models \exists < a > .\varphi$$

$$[\exists < a > .\varphi](X) \equiv \exists y, Y. Pref(Y) \wedge (X \uplus \{y\} = Y) \wedge P_a(y) \wedge \varphi(Y)$$

$$t, C \models \varphi \, Until \, \psi$$

$$[\varphi Until \psi](X) \equiv \exists Y. Pref(Y) \wedge X \subset Y \wedge \psi(Y) \wedge \forall Z. [Pref(Z) \wedge X \subseteq Z \subset Y \Rightarrow \varphi(Z)]$$

Basis of a set of Pomsets

The basis $B(\mathcal{L})$ contains the elements of $\mathcal{L}$ which are not a strict order extension of some other

$$B(\mathcal{L}) = \{u \in \mathcal{L} \mid \forall v \in \mathcal{L} : u \in OE(v) \Rightarrow u = v\}$$

$\mathcal{L}$ is basic if $\mathcal{L} = B(\mathcal{L})$

Weak and basic sets are in one-to-one correspondance

$$OE(\mathcal{L}) = OE(B(\mathcal{L}))$$

$$B(\mathcal{L}) = B(OE(\mathcal{L}))$$

Logics of pomsets: definability of representations

If $OE(\mathcal{L}) = L(\varphi)$ is FO/MSO definable then the set of step extensions $SE(\mathcal{L}) = OE(\mathcal{L}) \cap \mathbb{S}(\Sigma)$ is FO/MSO definable

$$SE(\mathcal{L}) = L(\varphi \bigwedge \forall x, y, z.(x\, co\, y \bigwedge x\, co\, z \bigwedge y \neq z) \Rightarrow y\, co\, z)$$

If $\mathcal{L} = L(\varphi)$ is FO/MSO definable then the base $B(\mathcal{L})$ is FO/MSO definable

$$B(\mathcal{L}) = L(\varphi \bigwedge \forall x, y.(x \lessdot y) \Rightarrow \neg\varphi_{x,y}) \text{ where } \varphi_{x,y} \text{ is } \varphi$$
$$[w \lessdot z/(w \lessdot z \bigwedge w \neq x \bigwedge z \neq y)]$$

Property to be used in the sequel

$B(\mathcal{L})$ is MSO def. $\Leftarrow \mathcal{L}$ is weak and MSO def. $\Rightarrow SE(\mathcal{L})$ is MSO def.

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Step transition systems (Definition)

An STS on the alphabet Σ: a tuple $\mathcal{A} = (Q, \rightarrow, q_{in}, F)$ where

Q set of states, $q_{in} \in Q$ initial state, $F \subseteq Q$ set of final states

$\rightarrow \subseteq Q \times M(\Sigma) \times Q$ transition relation : for any $q, q' \in Q$ and $m, m_1, m_2 \in M(\Sigma)$:

$$q \stackrel{\emptyset}{\longrightarrow} q' \Rightarrow q = q'$$

$$m = m_1 \oplus m_2 \wedge q \stackrel{m}{\longrightarrow} q' \Rightarrow \exists q_1 : q \stackrel{m_1}{\longrightarrow} q_1 \stackrel{m_2}{\longrightarrow} q'$$

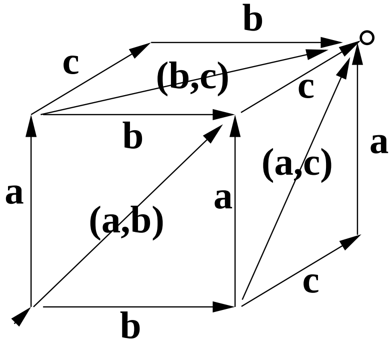
Notation

$$q \stackrel{u_1.u_2}{\longrightarrow} q' \text{ iff } \exists q_1 : q \stackrel{u_1}{\longrightarrow} q_1 \stackrel{u_2}{\longrightarrow} q' \text{ for } u_1, u_2 \in M(\Sigma)^*$$

step semantics of STS

The step language $SL(\mathcal{A})$ of A is

$$SL(\mathcal{A}) = \{u \in M(\Sigma)^\star | \exists q \in F : q_{in} \xrightarrow{u} q\}$$



$$SL(\mathcal{A}) = \{a.b.c,\ a.c.b,\ b.a.c,\ b.c.a,\ (a \parallel b).c\ ,\ a.(b \parallel c),\ b.(a \parallel c)\}$$

Pomset semantics of STS 1: Step closure

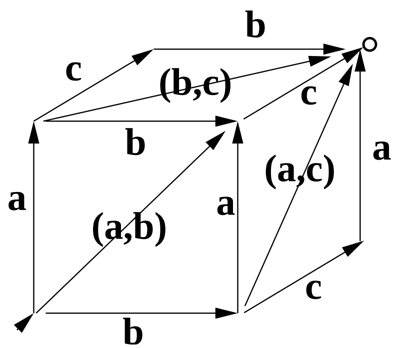
The pomset language $L(\mathcal{A})$ of $\mathcal{A}$ is the step-closure of $SL(\mathcal{A})$, i.e. it contains all pomsets t whose step extensions are in $SL(\mathcal{A})$

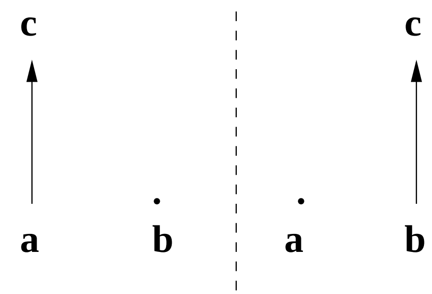
$$L(\mathcal{A}) = \{t \in \mathbb{P}(\Sigma) | SE(t) \subseteq SL(\mathcal{A})\}$$

Note that $SE(L(\mathcal{A})) = SL(\mathcal{A})$ and that $L(\mathcal{A})$ is step-closed:

for all $t \in \mathbb{P}(\Sigma)$, $SE(t) \subseteq SE(L(\mathcal{A})) \Leftrightarrow t \in L(\mathcal{A})$

Step transition systems





Step language : $SL(\mathcal{A}) = \{a.b.c,\ a.c.b,\ b.a.c,\ b.c.a,\ (a \parallel b).c\ ,\ a.(b \parallel c),\ b.(a \parallel c)\}$

$(a.c) \parallel b$ and $a \parallel (b.c)$ belong to the step closure of $SL(\mathcal{A})$:

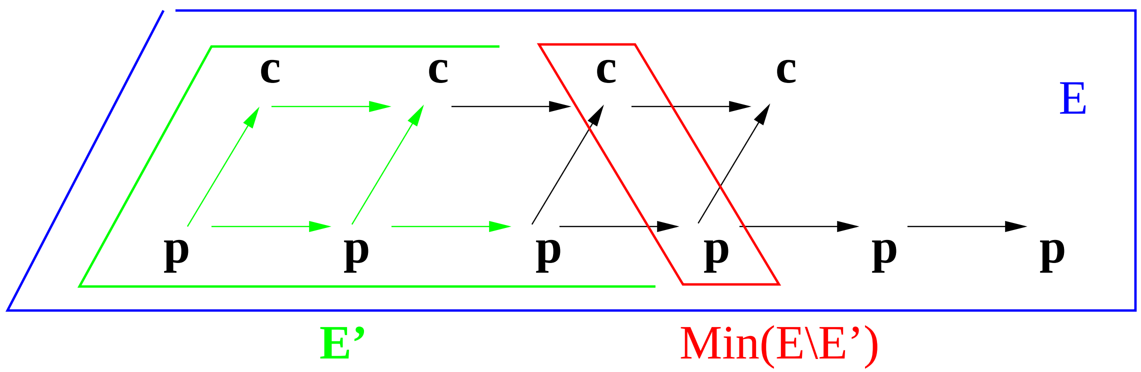
$$SE((a.c) \parallel b) = \{a.b.c,\ a.c.b,\ b.a.c, (a \parallel b).c\ ,\ a.(b \parallel c)\}$$

Pomset language : $L(\mathcal{A}) = SL(\mathcal{A}) \cup \{(a.c) \parallel b\ ,\ a \parallel (b.c)\}$

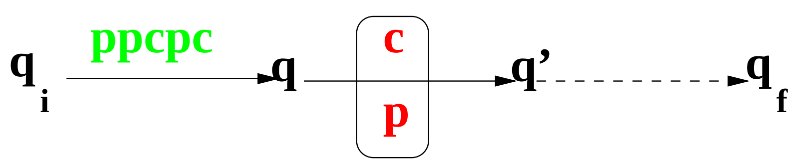
Basis of the pomset language : $B(L(\mathcal{A})) = \{(a.c) \parallel b,\ a \parallel (b.c)\}$

Pomset semantics of STS 2: processes

Petri Nets firing pomsets, Local trace pomsets [Kuske,Morin]



$t = (E, \preceq, l)$, prefix $t' = t/E'$, $(p, c) = l(Min_{\preceq}(E \setminus E'))$, $u = ppcpc \in LE(t')$,



Pomset semantics of STS 2: processes

$t = (E, \preceq, l)$ is a process of $\mathcal{A}$, $t \in \wp(\mathcal{A})$, iff

for *any* prefix $t' = (E', \preceq / E', l/E')$ of t and *any* $u \in LE(t')$, then

$$q_{in} \xrightarrow{u} q \xrightarrow{m} q' \longrightarrow q_f. \text{ where } m = l(Min_{\preceq}(E \setminus E'))$$

t is a final process, $t \in \wp_f(\mathcal{A})$, if furthermore $LE(t) \subseteq SL(\mathcal{A})$

Lemma: for any STS $\mathcal{A}$

$$Pref(L(\mathcal{A})) \subseteq \wp(\mathcal{A}) \text{ and } L(\mathcal{A}) \subseteq \wp_f(\mathcal{A})$$

For Deterministic STS, the two semantics coincide

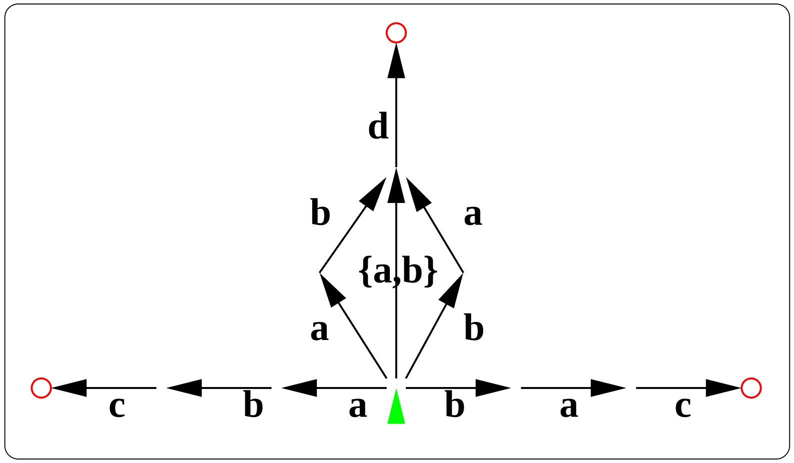
$\mathcal{A}$ is deterministic if for any $q, q', q'' \in Q$ and $m \in M(\Sigma)$:

$$q \xrightarrow{m} q' \wedge q \xrightarrow{m} q'' \Rightarrow q' = q''$$

Theorem:

If $\mathcal{A}$ is deterministic, $\wp(\mathcal{A}) = Pref(L(\mathcal{A}))$ and $\wp_f(\mathcal{A}) = L(\mathcal{A})$

For non deterministis STS, the two semantics may differ



$(a \parallel b).c$ is a final process of A ... but is not even in the step language

$SL(\mathcal{A}) = SE\{(a \parallel b).d, a.b.c, b.a.c\} = L(\mathcal{A})$

Step closure appropriate semantics for general STS.

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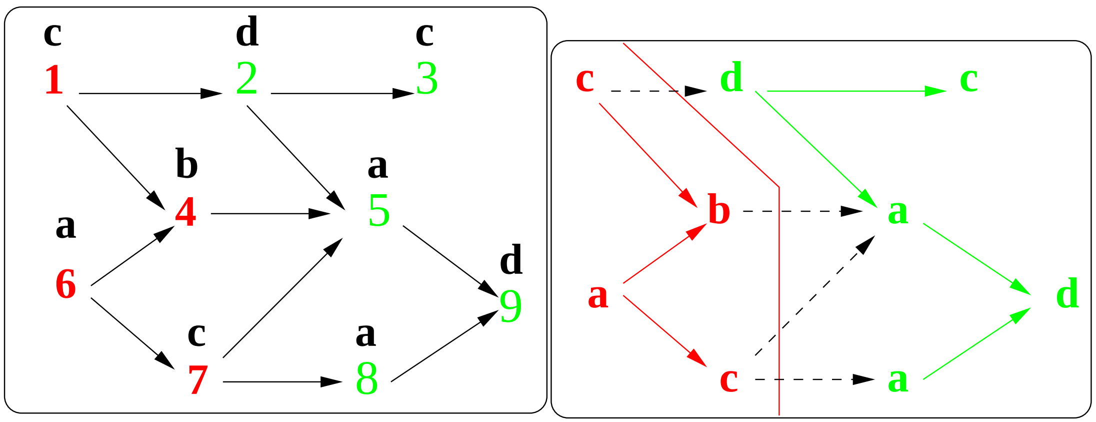
Prefixes and Residues



An ideal Prefix Residue

An ideal and its complement underly a prefix and a residue

In case of autoconcurrency, residues are not uniquely defined: $a \parallel b$ and $a.b$ are residues of $a \parallel (a.b)$ after a

The residue of r after u is a set denoted r/u

$$\mathcal{L}/u = \{v \in \mathbb{P}(\Sigma) | \exists r \in \mathcal{L} : v \in r/u\}$$

Languages of Deterministic STS

Remark: if $\mathcal{A}$ is deterministic

for any $u, v \in M(\Sigma)^\star$, such that $q \xrightarrow{u} q'$ and $q \xrightarrow{v} q"$,

if $LE(u) \cap LE(v) \neq \oslash$, then $q' = q"$

Lemma:

$K \subseteq M(\Sigma)^\star$ is the step language of a DSTS iff $SE(K) = K$ and

for any $u, v \in Pref(K)$, $LE(u) \cap LE(v) \neq \oslash \Rightarrow K/u = K/v$

A characterization of DSTS languages

$\mathcal{L} \subseteq \mathbb{P}^{nac}(\Sigma)$ is consistent [Arnold] :

for any $u, v \in Pref(\mathcal{L})$:$LE(u) \cap LE(v) \neq \oslash \Rightarrow u = v$

$\mathcal{L} \subseteq \mathbb{P}(\Sigma)$ is quasi-consistent

for any $u, v \in Pref(\mathcal{L})$: $LE(u) \cap LE(v) \neq \oslash \Rightarrow \mathcal{L}/u = \mathcal{L}/v$

Theorem:

$\mathcal{L} \subseteq \mathbb{P}(\Sigma)$ is the pomset language of a Deterministic STS iff it is step closed and quasi-consistant

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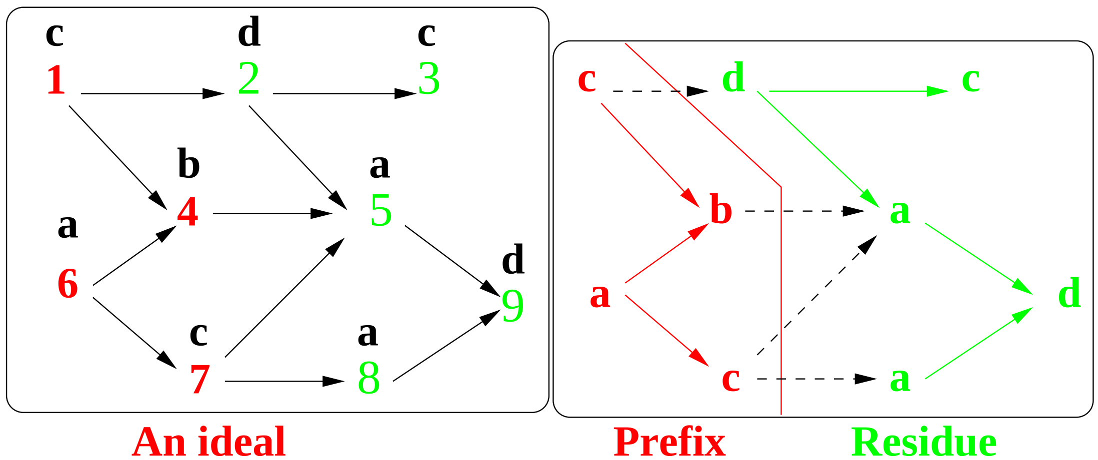
Regular sets of pomsets: Prefixes and Residues



An ideal and its complement underly a prefix and a residue

In case of autoconcurrency, residues are not uniquely defined: $a \parallel b$ and $a.b$ are residues of $a \parallel (a.b)$ after a

The residue of r after u is a set denoted r/u

Regular sets of pomsets [F,Morin02]

the residue of $\mathcal{L} \subseteq \mathbb{P}(\Sigma)$ after u is $\mathcal{L}/u = \{v \in \mathbb{P}(\Sigma) | \exists r \in \mathcal{L} : v \in r/u\}$

the residue equivalence $\simeq_L$ induced by $\mathcal{L}$ is $u \simeq_L v \overset{def}{\equiv} \mathcal{L}/u = \mathcal{L}/v$

$\mathcal{L}$ is regular iff $\simeq_L$ is finite

Coincides with usual definition for words, coincides with the regularity of $LE(\mathcal{L})$ for traces and MSC.

Properties of regular sets of pomsets [F,Morin02] [F05]

Closed for union, concatenation, iteration, parallel composition

Closed for image and reverse image through renaming and projection

$\mathcal{L}$ regular $\Rightarrow$ $OE(\mathcal{L})$ regular $\Rightarrow$ $SE(\mathcal{L})$ regular $\Rightarrow$ $LE(\mathcal{L})$ regular

If $\mathcal{L}$ is step-closed: $\mathcal{L}$ regular $\Leftrightarrow$ $SE(\mathcal{L})$ regular

Languages of finite DSTS

Theorem : Let $\mathcal{L} \subseteq \mathbb{P}(\Sigma)$ the following are equivalent:

- $\mathcal{L}$ is the pomset language of a finite DSTS
- $\mathcal{L}$ is regular, step-closed, width-bounded and quasi-consistent

Lemma :

If $\mathcal{L}$ is a finite DSTS language then $LE(\mathcal{L})$ is regular and for all $m \in M(\Sigma)$, $\mathcal{L}(m)$ is regular

where $\mathcal{L}(m)$ is the sets of words $u \in \Sigma^\star$ s.t. $u.m \in Pref(\mathcal{L})$

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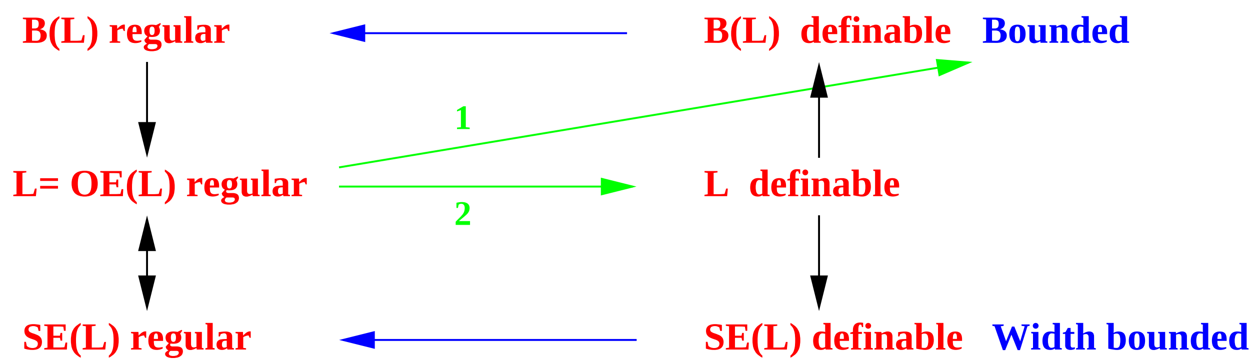
Relating regularity and MSO definability of DSTS languages

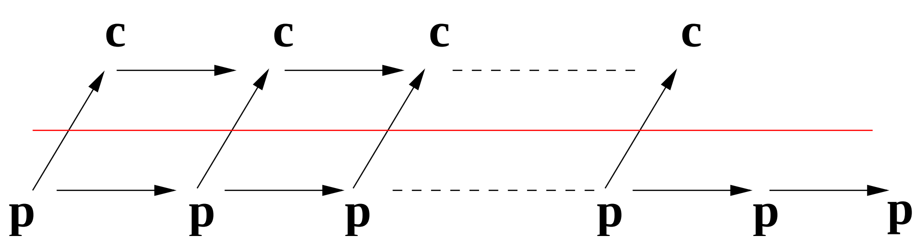
A scheme of properties for step closed languages and additional properties for DSTS languages.



Blue arrows : a property of “prime bounded” sets of pomsets.

Prime bounded sets of pomsets [Kuske 98]

A pomset t is k-prime-bounded if any prefix "cuts" at most k prime intervals.



Equivalently: $t = (E, \preceq, l)$ is k-prime-bounded if it has a k-chains covering, i.e. a mapping $\lambda : E \longrightarrow 2^{[k]}$: such that $\forall e, e' \in E$,

$$e \lessdot e' \Rightarrow \lambda(e) \cap \lambda(e') \neq \emptyset \text{ and } e\, co\, e' \Rightarrow \lambda(e) \cap \lambda(e') = \emptyset$$

Theorem [F05]:

If $\mathcal{L} \subseteq \mathbb{P}(\Sigma)$ is prime bounded and MSO definable then $\mathcal{L}$ is regular

Trace monoid $M(\Sigma \times 2^{[k]}, \|)$, projection $\pi : \Sigma \times 2^{[k]} \longrightarrow \Sigma$, then $\mathcal{L}' = \pi^{-1}(\mathcal{L}) \cap M(\Sigma \times 2^{[k]}, \|)$ is a definable subset of $M(\Sigma \times 2^{[k]}, \|)$, and thus regular. Furthermore $\mathcal{L} = \pi(\mathcal{L}')$.

MSO definable and width-bounded step-closed languages are regular.

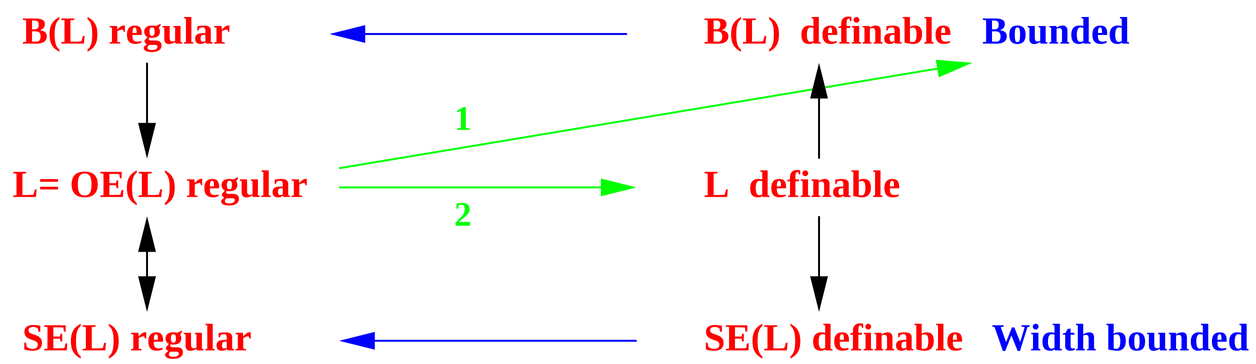
Theorem [F05]: Let $\mathcal{L}$ be a width-bounded and step-closed language, then $\mathcal{L}$ MSO definable $\Rightarrow$ $\mathcal{L}$ regular

Proof :

$\mathcal{L}$ MSO definable $\Rightarrow$ $SE(\mathcal{L})$ MSO definable $\Rightarrow$ $SE(\mathcal{L})$ regular $\Rightarrow$ $\mathcal{L}$ regular

1. If $\mathcal{L}$ is step-closed, it is weak. If furthermore $\mathcal{L}$ is MSO definable, then $SE(\mathcal{L}) = \mathcal{L} \cap \mathbb{S}(\Sigma)$ is MSO definable too
2. If $\mathcal{L}$ is width-bounded by k , then $SE(\mathcal{L})$ is prime bounded by k^2 . $SE(\mathcal{L})$ being MSO definable then $SE(\mathcal{L})$ is regular.
3. $\mathcal{L}$ is step closed thus $SE(\mathcal{L})$ regular $\Rightarrow$ $\mathcal{L}$ regular .

Relating regularity and MSO definability of DSTS languages



Proof of 1) Result of [KM00] for pomsets without autoconcurrency, extended to width-bounded sets of pomsets.

Proof of 2) Using a technique from graph theory [Courcelle]

The basis of a finite DSTS language is prime bounded

If $t = (E, \preceq, l)$ has width $\leq k$, then t has a k-chains partition:

a partition of E, $C = \{D_1, ..., D_k\}$, $E = \uplus_{0<i\leq k} D_i$, where each D_i is a chain (i.e. totally ordered)

This is a mapping $\lambda : E \rightarrow [k]$ such that $\forall e, e' \in E,\ e\, co\, e' \Rightarrow \lambda(e) \neq \lambda(e')$

Let $\mathcal{L} \subseteq \mathbb{P}_{w<k}(\Sigma)$ and $\pi : \Sigma \times [k] \longrightarrow \Sigma$ the 1st projection, then

$$\mathcal{L}' = \pi^{-1}(\mathcal{L}) \cap \mathbb{P}^{nac}(\Sigma \times [k])$$

is the language of a finite DSTS without autoconcurrency, such that $\mathcal{L} = \pi(\mathcal{L}')$. By a result of [KM00] $B(\mathcal{L}')$ is prime-bounded, and so is $B(\mathcal{L})$.

Languages of finite DSTS are MSO definable

Usefull Lemma:

$t = (E, \leq, l) \in \mathcal{L} = L(\mathcal{A})$ iff $u \in LE(\mathcal{L})$ for at least one $u \in LE(t)$ and

for any prefix $t' = (E', \leq /E', l/E')$ of t , $u \in \mathcal{L}(m)$ for at least one $u \in LE(t')$, where $m = l(Min_{\leq}(E \setminus E')).\Box$

If $\mathcal{A}$ is finite then $LE(\mathcal{L})$ is regular and $\mathcal{L}(m)$ are regular for all $m \in M_k(\Sigma)$. There are MSO formulae ψ_L, and $\{\psi_m, m \in M_k(\Sigma)\}$ such that $u \in LE(\mathcal{L}) \Leftrightarrow u \models \psi_L$ and $u \in \mathcal{L}(m) \Leftrightarrow u \models \psi_m$.

Key point: construct from any formula ψ on words a formula $\widehat{\psi}$ such that for all pomsets t of interest, $t \models \widehat{\psi}$ if and only if $u \models \psi$ for some $u \in LE(t')$. We rely on the width bound of $\mathcal{L}$.

A parameterized transduction from pomsets to words (1)

k chains partitions are MSO definable

$$ChainPart(X_1...X_k) \equiv Partition(X_1...X_k) \bigwedge \forall x, y. \bigvee_{i\in[k]}(x \in X_i) \wedge [\bigwedge_{i\in[k]}(x \in X_i \wedge y \in X_i \Rightarrow x \preceq y \vee y \preceq x)]$$

Theorem [Courcelle 96]:

$\exists$ MSO formula $\theta_k(x, y, X_1, ..., X_k)$ s.t. for any $t \in \mathbb{P}_{w<k}(\Sigma)$, $t = (E, \preceq_t, l)$, any k chains partition $C = \{D_1, ..., D_k\}$ of t:

$$(E, \preceq_C, l) : e \preceq_C e' \Leftrightarrow (t, e, e', D_1, ..., D_k) \models \theta_k(x, y, X_1, ..., X_k).$$

is a linear extension of t , which we denote by $LinE_k(t, D_1, ..., D_k)$.

A parameterized transduction from pomsets to words (2)

Let $\psi \in MSO(\Sigma, \preceq)$ be a closed formula, replace the occurrences of $x \preceq y$ in ψ by $\theta_k(x, y, X_1, ..., X_k)$ to obtain a formula $\overline{\psi}(X_1, ..., X_k)$ s.t.

$$(t, D_1, ..., D_k) \models \overline{\psi} \text{ iff } LinE_k(t, D_1, ..., D_k) \models \psi.$$

The formula $\widehat{\psi} \stackrel{def}{=} [\forall X_1, ..., X_k.ChainPart(X_1, ..., X_k) \Rightarrow \overline{\psi}(X_1, ..., X_k)]$ is such that for any $t = (E, \preceq, l)$

$$t \models \widehat{\psi} \text{ if and only if for any } D_1, ..., D_k \subseteq E :$$
$$(t, D_1, ..., D_k) \models ChainPart(X_1, ..., X_k) \text{ implies } LinE_k(t, D_1, ..., D_k) \models \psi$$

Languages of finite DSTS are MSO definable (end)

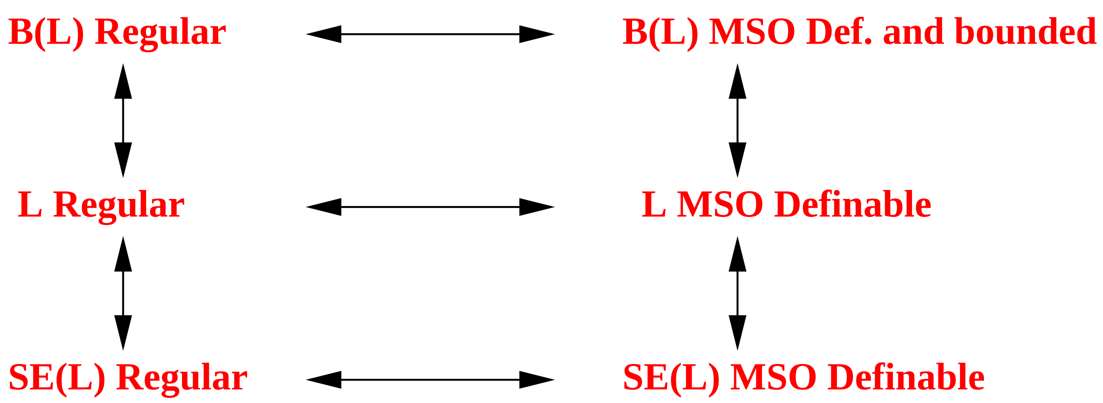
Let $\mathcal{A}$ be a finite DSTS and $\mathcal{L} = L(\mathcal{A})$, let ψ_L and $\{\psi_m, m \in M_k(\Sigma)\}$ be the MSO formulae such that $u \in LE(\mathcal{L}) \Leftrightarrow u \models \psi_L$ and $u \in \mathcal{L}(m) \Leftrightarrow u \models \psi_m$.

We can build from the induced formulae $\widehat{\psi_L}$ and $\{\widehat{\psi_m}, m \in M_k(\Sigma)\}$ an MSO formula ψ_A such that for any $t \in \mathbb{P}_{w<k}(\Sigma)$, $t \in \mathcal{L}$ if and only if $t \models \psi_A$.

A resulting scheme for DSTS languages.

Theorem: Let $\mathcal{L}$ be width bounded, step closed and quasi consistent then

$\mathcal{L}$ is the language of a finite DSTS iff it satisfies one of the following equivalent conditions:



Outline

Pomsets and representations

Logic of pomsets

Step transition systems and their languages

A characterization of DSTS languages

Regular DSTS languages

Relating MSO definability and regularity

Petri Nets languages

Conclusion

Petri Nets languages

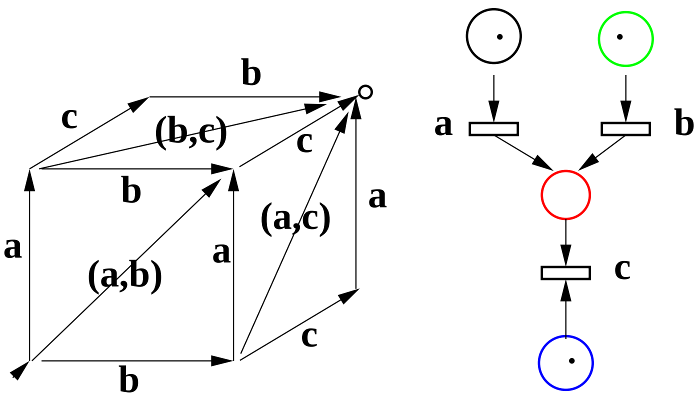
Two main concurrent semantics of Petri nets:

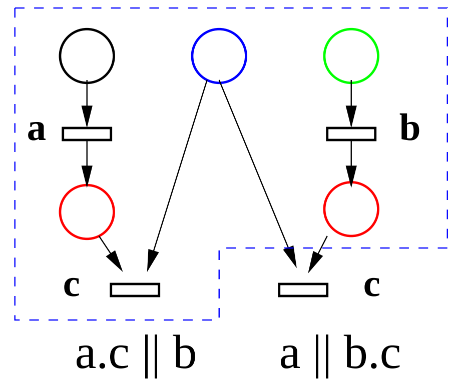
Individual tokens : "Unfolding pomsets" are conflict free prefixes of net unfoldings (occurrence nets), places represent a single token in some place of the net. [Goltz 84] [Best,Devillers 87] etc

Collective tokens : "Firing pomsets" which can be derived from their step marking graph [Grabowski 81] etc.. coincide with STS processes

By a theorem of [Kiehn 88][Vogler 91], both resulting sets of pomsets have the same basis

Induced properties for Petri Nets languages





$L(\mathcal{A})$ is the set of firing pomsets of $\mathcal{N}$

$B(L(\mathcal{A})) = \{(a.c) \parallel b,\ a \parallel (b.c)\}$ is the basis of the set of unfolding pomsets of $\mathcal{N}$:

Conclusion

A simple characterization of DSTS pomset languages, introduces autoconcurrency, extends consistent sets and local traces.

Büchi-like properties: e.g. for bounded-width DSTS languages, MSO definability and regularity coincide.

Applies to the basis of unfolding pomsets of Petri Nets: regularity coincides with MSO-definability and prime-boundedness.

Perspectives

Enlarge to step closed sets of pomsets (non-deterministic STS)

Composition operators : syntactic characterisations

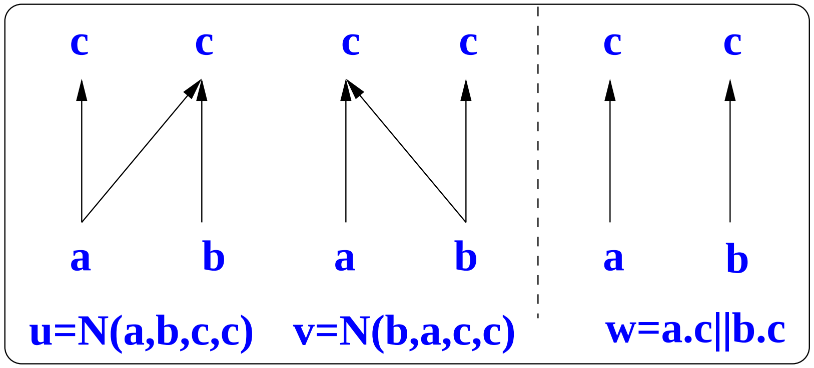
Abstractions, projections and refinements

Effectiveness and complexity of formulae transformations , Model-checking

From STS to Event Structures

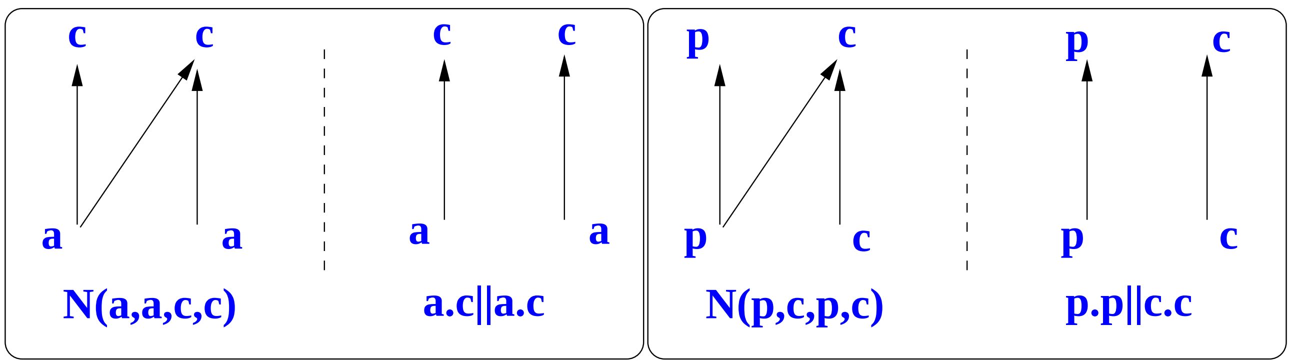
Merci de votre attention

Step extensions and step-closure

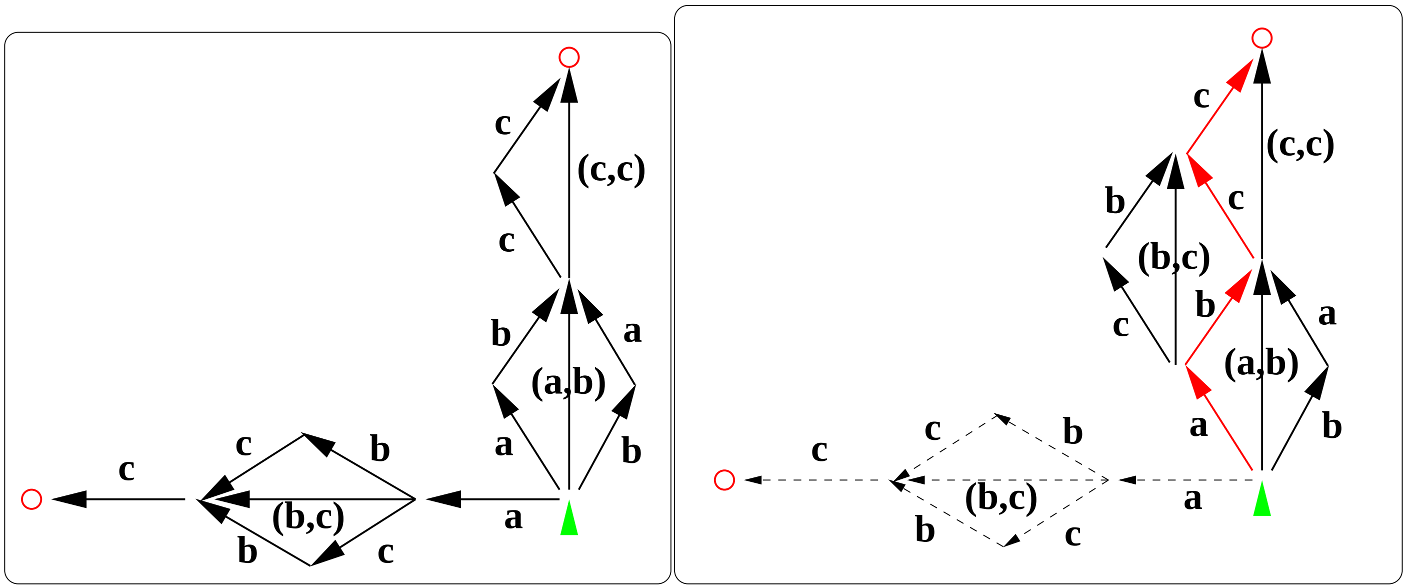


$$SE(w) = SE(\{u, v\}) = SE(\{(a||b).(c||c), a.(b||c).c, b.(a||c).c\})$$

Step extensions and step-closure



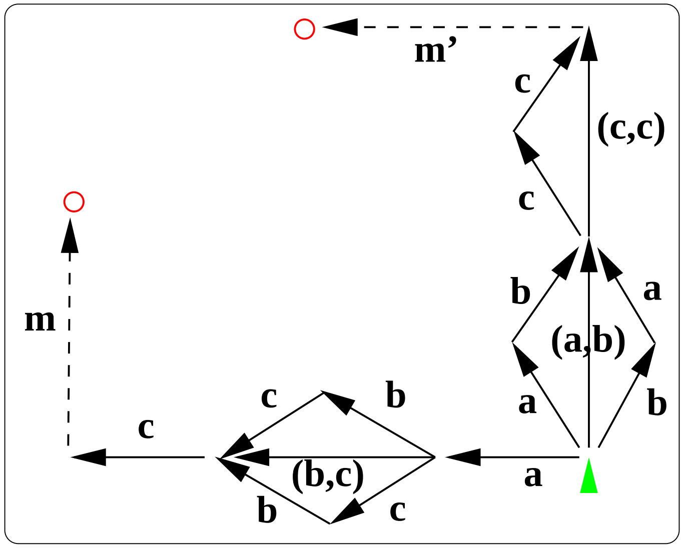
Step transition systems : example



Step language $SL(\mathcal{A}) = SE\{(a \parallel b).(c \parallel c), a.(b \parallel c).c\} = SE(N(a,b,c,c))$

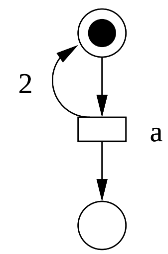
Pomset language $L(\mathcal{A}) = OE(N(a,b,c,c)),\ Base(L(\mathcal{A})) = N(a,b,c,c)$

Step transition systems , Determinism vs non determinism 2



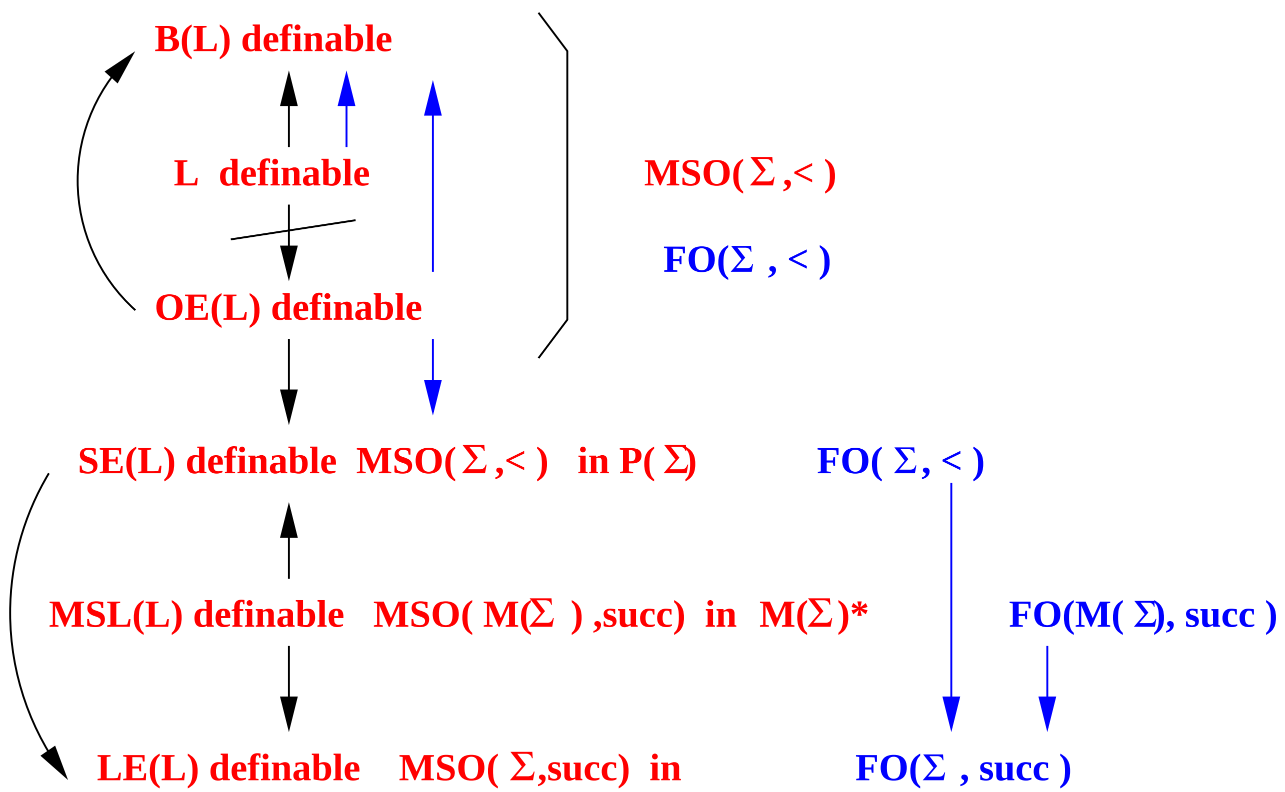
For non-deterministic STS, $Pref(L(\mathcal{A}))$ is not step-closed: $N(a, b, c, c)$ not in $Pref(L(\mathcal{A}))$

Regularity of $LE(\mathcal{L}) \not\Rightarrow$ Regularity of $SE(\mathcal{L})$

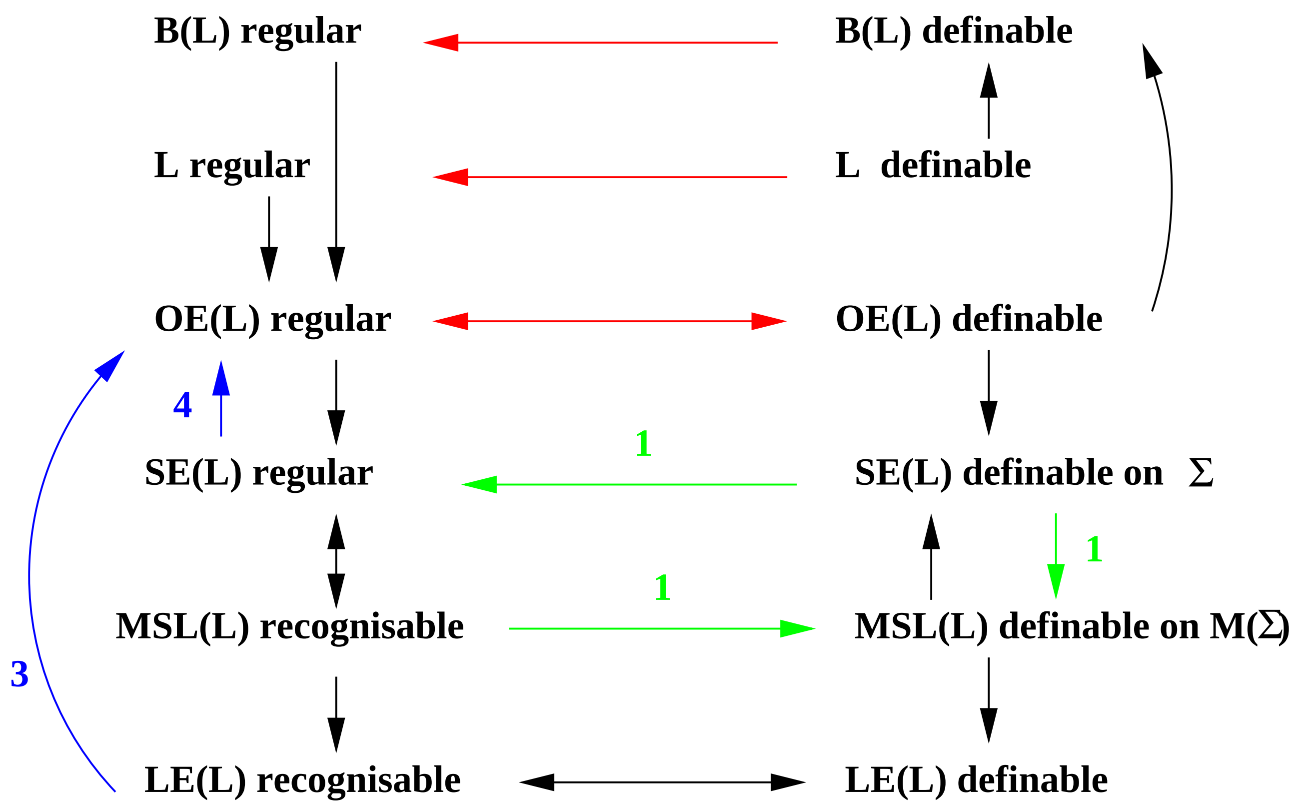


$LE(L(N)) = a^\star$ is regular , $SE(L(N)) \supseteq \{a.(2 \times a)....(2i \times a)\}$ is not regular

Note that regular Petri nets languages (with initial marking) are width-bounded.


B(L) definable
L definable
OE(L) definable
MSO(Σ,<)
FO(Σ , <)
SE(L) definable MSO(Σ,<) in P(Σ)
FO(Σ, <)
MSL(L) definable MSO(M(Σ) ,succ) in M(Σ)*
FO(M(Σ), succ)
LE(L) definable MSO(Σ,succ) in
FO(Σ , succ)

Regularity vs MSO definability [F05]



1: L width bounded

2: prime bounded

3: OE(L) linear closed

4: OE(L) step closed